Thermochimica Acta 512 (2011) 208–216

Contents lists available at ScienceDirect

Thermochimica Acta

journal homepage: www.elsevier.com/locate/tca

An analytical thermal resistance model for calculating mean die temperature of a typical BGA packaging

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ARTICLE INFO

Article history:
Received 30 April 2010
Received in revised form 18 October 2010
Accepted 20 October 2010
Available online 3 November 2010

Keywords:
Analytical solution
Thermal resistance model
Mean die temperature

ABSTRACT

An analytical thermal resistance network model is developed for calculating mean die temperature of a typical BGA packaging. The thermal resistance network is established based on heat dissipation paths from die to ambient. Every thermal resistance in the network can be calculated by analytical expressions. The proposed model is applied to a typical BGA packaging. Simulations to obtain the die temperature of the packaging were also done by COMSOL. The data comparison shows that the mean die temperatures calculated by the present model are very close to the ones obtained by simulations. It is demonstrated that the proposed model can be used to predict the mean die temperature of BGA packaging accurately. This proposed model is simple and resource-saving for the semi-conductor industry to predict the mean die temperature of typical BGA packaging and also it provides an optimization method to choose materials for good thermal management of BGA packaging.

1. Introduction

Ball Grid Array (BGA) packaging is one of the most commonly used packaging types applied for microelectronic devices. To guarantee the reliability and longevity of BGA productions, it is necessary and important to analyze thermal characterization of BGA packaging, especially the die temperature of BGA packaging. In the semiconductor industry, numerical simulations and experiments are the mostly used methods to obtain the die temperature of a chip packaging. These methods commonly cost much time and resources. To reduce the time of obtaining die temperature and improve the efficiency of microelectronic device design, it is of significance to establish an analytical thermal model for analyzing and calculating die temperature of chip packaging.

Traditionally, thermal resistance is defined as:

$$R_{\mathrm{th}} = \frac{T - T_{\mathrm{ref}}}{Q} \quad (1)$$

where T is some critical temperature (usually the junction temperature), T_{ref} is reference temperature, and Q is usually the steady-state heat dissipation of the component.

Since chip packaging usually contains multilayers, the structure is complex and the ambient conditions are variable, there is no such analytical single thermal resistance model can be used to calculate the die temperature until today. Bar-Cohen et al. [1] firstly proposed a thermal resistance network model to calculate junction temperatures. The network topology is star-shaped and only under isothermal condition can it predict the junction temperature accurately. Europe [2,3] carried out a 3-year European collaborative project, named DELPHI, whose goal is to find solutions to predict the operating temperatures of critical electronic parts at the component-, board- and system-level. In the project DEPHI, Lasance et al. [4] established an improved star-shaped compact thermal resistance network model which was boundary condition independent. Surface-to-surface resistors were added to the improved star-shaped network for better representing realistic characterization of heat transfer. In all of the 38 kinds of different conditions mentioned in Ref. [4], the improved model can predict the junction temperature accurately, the error is only 1–2%. Aranyosi et al. [5] used the thermal model presented in Ref. [4] to analyze conduction cooled electronic applications. Two general network topologies, incorporating both simple star-shaped network mentioned in Ref. [1] and more complex, shunted network, were developed. They found that optimized star-shaped compact thermal model predicted the junction temperature accurately. Considering the complexity of multi-resistor thermal network models like the one in Ref. [4] and unavailability of the required information about the packaging internal structure and materials for end-users, Tal and Nabi [6] put forward an analytic method for con-

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doi:10.1016/j.tca.2010.10.009

Nomenclature

a	half length of die [m]
a_1	half length of rectangular heat source s1 [m]
$A_{\mathrm{contact\ plane}}$	area of contact plane [m^2]
A_{Pt}	area of top surface of PCB [m^2]
A_{Pbo}	area of bottom surface of PCB [m^2]
A_{plane}	area of plane [m^2]
A_{source}	area of heat source [m^2]
A_{subb}	area of bottom surface of substrate [m^2]
A_0, A_m, A_n, A_{mn}	Fourier coefficients
b	half width of die [m]
b_1	half width of rectangular heat source s1 [m]
c	half length of BGA packaging [m]
c_1	half length of PCB [m]
d	half width of BGA packaging [m]
d_1	half width of PCB [m]
c_2, c_3, d_2, d_3	dimensions of heat source s2 [m]
D_1	diameter of top surface circle of truncated cone [m]
D_2	diameter of bottom surface circle of truncated cone [m]
h_{subbo}	heat transfer coefficient at bottom substrate exposed to ambient [W/(m^2 K)]
h_{me}	heat transfer coefficient at edge of mold compound [W/(m^2 K)]
h_{equ}	equivalent heat transfer coefficient at bottom surface of substrate [W/(m^2 K)]
h_{Pbo}	heat transfer coefficient at bottom surface of PCB [W/(m^2 K)]
h_{Pt}	heat transfer coefficient at top surface of PCB [W/(m^2 K)]
h_{mt}	heat transfer coefficient at top surface of mold compound [W/(m^2 K)]
k_{b}	thermal conductivity of solder balls [W/(m K)]
k_{m}	thermal conductivity of mold compound [W/(m K)]
k_{P}	thermal conductivity of PCB [W/(m K)]
k_{sub}	thermal conductivity of substrate [W/(m K)]
L	height of truncated cone [m]
L_i	length of ith rectangular heat source [m]
m, n	indices of summations
N	number of solder balls
q	heat flux of heat source s1 and s2 [W/m^2]
q_{suba}	heat flows from substrate to solder balls and ambient [W]
Q	heat flow rate of die or heat source [W]
R_{1Dmd}	one-dimension thermal conduction resistance of die size mold compound [K/W]
R_{1DP}	one-dimensional thermal conduction resistance of PCB [K/W]
R_{1Dsub}	one-dimensional thermal conduction resistance of substrate [K/W]
R_{b}	thermal resistance of solder balls [K/W]
R_{b1}	thermal resistance of a solder ball [K/W]
R_{hPt}	thermal resistance between top surface of PCB and ambient [K/W]
R_{hPbo}	thermal resistance between bottom surface of PCB and ambient [K/W]
R_{ma}	thermal resistance of mold compound and the part between mold compound and ambient [K/W]
R_{Pa}	thermal resistance of PCB and the part between PCB to ambient [K/W]
R_{Ps}	spreading resistance of PCB [K/W]
R_{s}	spreading resistance [K/W]
R_{subs}	spreading resistance of substrate [K/W]
R_{sub}	thermal resistance of substrate [K/W]
R_{suba}	thermal resistance between bottom surface not covered by balls of substrate and ambient [K/W]
R_{th}	thermal resistance [K/W]
R_{to}	total thermal resistance from die to ambient [K/W]
R_{tot}	total thermal resistance of top part of BGA [K/W]
R_{total}	total thermal resistance of multiple heat sources [K/W]
t_{d}	thickness of die [m]
t_{m}	thickness of mold compound [m]
t_{sub}	thickness of substrate [m]
t_{P}	thickness of PCB [m]
T	some critical temperature [K]
T_{a}	temperature of ambient [K]
$T_{\mathrm{contact\ plane}}$	temperature of contact plane [K]
$\overline{T}_{\mathrm{contact\ plane}}$	mean temperature of contact plane [K]
$\overline{T}_{\mathrm{d\ model}}$	mean temperature of die calculated by present model [K]
$\overline{T}_{\mathrm{dsimulation}}$	mean temperature of die obtained by simulation [K]
T_{ref}	reference temperature [K]
T_{source}	source temperature [K]
$\overline{T}_{\mathrm{source}}$	mean temperature of heat source [K]
$\overline{T}_{\mathrm{subb}}$	mean temperature of bottom surface of substrate [K]
W_i	width of ith heat source [m]
xi, yi ith	heat source centroid [m]
$x_{\mathrm{BGA}}, y_{\mathrm{BGA}}$	substitute rectangular centroid [m]

Greek symbols

θ_i	temperature excess distribution on top surface of PCB for ith heat source [K]
$\overline{\theta}$	mean temperature excess of substitute rectangular area [K]
δ_m	eigenvalues, $m\pi/c$ or $m\pi/c_1$
λ_n	eigenvalues, $n\pi/d$ or $n\pi/d_1$
δ	eigenvalues, $n\pi/2d_1$
λ	eigenvalues, $m\pi/2c_1$
β	eigenvalues, $\equiv \sqrt{\delta^2+\lambda^2}$
β_{mn}	eigenvalues, $\equiv \sqrt{\left(\frac{\delta_{xm}}{c}\right)^2+\left(\frac{\delta_{yn}}{d}\right)^2}$ or $\sqrt{\delta_m^2+\lambda_n^2}$
δ_{xm}, δ_{yn}	eigenvalues
ϕ, φ	spreading function
ξ	dummy variable

Subscripts

a	ambient
b	ball
bo	bottom
b1	one ball
d	die
1D	one-dimension
equ	equivalent
i	index denoting heat source 1–5
m	mold compound
P	PCB
t	top
th	thermal
to	total
ref	reference
sub	substrate
s	spreading

Superscript

$\overline{(\bullet)}$	mean value

verting standardized IC-packaging thermal resistances, junction to ambient resistance (R_{ja}) and junction to case resistance (R_{jc}), into a two-resistor thermal model which contains a junction to top resistance (R_{jt}) and a junction to board resistance (R_{jb}). With the help of the method named PERIMA, one can evaluate the two resistances in the two-resistor model using an analytic algorithm. Although lacking of the information about packaging, this method can evaluate resistances in a reasonable range. Garcia and Chiu [7] used two-resistor model to analyze multiple-die and multi-chip packaging. They concluded that chip-scale packaging was suitable for two-resistor thermal model because of small thermal gradients and low thermal resistance between the die stack.

The die in packaging is commonly smaller than the substrate or heat spreader, which the die is located on. In addition, the area of PCB, where packaging is bonded on, is much more than the one of packaging. Therefore, there exists spreading resistance as heat flows from die into heat spreader or from packaging to PCB. The spreading resistance was obtained by means of the definition proposed by Mikic and Rohsenow [8]:

$$R_{\mathrm{s}} = \frac{\bar{T}_{\mathrm{source}} - \bar{T}_{\mathrm{contact\_plane}}}{Q} \tag{2}$$

The mean temperature of the heat source area is obtained from:

$$\bar{T}_{\mathrm{source}} = \frac{1}{A_{\mathrm{source}}} \int_{A_{\mathrm{source}}} T_{\mathrm{source}} dA_{\mathrm{source}} \tag{3}$$

And the mean temperature of the contact plane is obtained from:

$$\bar{T}_{\mathrm{contact\_plane}} = \frac{1}{A_{\mathrm{contact\_plane}}} \int_{A_{\mathrm{contact\_plane}}} T_{\mathrm{contact\_plane}} dA_{\mathrm{contact\_plane}} \tag{4}$$

Spreading resistance is much more than one-dimension conduction resistance. Thus spreading resistance is the major thermal resistance when heat transfers from die to substrate or heat spreader, as well as when heat dissipates from packaging to PCB in steady state. Yovanovich et al. did series of researches on spreading resistance and obtained many analytical solutions for calculating spreading resistances in different situations. Yovanovich et al. [9] analyzed spreading resistance of isoflux rectangular and strips on compound flux channels and obtained general spreading resistance expression. They also studied spreading resistance in compound and orthotropic systems and also found analytical solutions [10,11]. In Refs. [12–14], Yovanovich et al. conducted research on spreading resistance in rectangular flux channel and obtained spreading resistance expressions based on different boundary conditions. Moreover, they discussed the influence of geometry and edge cooling on spreading resistance.

There are many other methods used for evaluating junction temperatures besides thermal resistance network models. One method uses the superposition method. Lall et al. [15] presented superposition method for evaluating junction temperatures of multichip modules. Although this method needs few experiments, it is very accurate for a model with equal die size and symmetric die location. Zahn [16] presented the linear superposition theory on the non-linear matrix multiplier. This theory can be applied to a small multiple-output device working over a wide range of operating power at a natural convection steady state environment only. Another method is RSM (response surface method). As explained by Roux, RSM is a method for constructing global approximations of a system's behavior based on the results calculated at various points in the design space. Zahn [17] also discussed the RSM for the thermal characterization of multiple heat source packaging. Accurate results can be obtained using RSM, but it requires many experiments to establish the response surface model. In addition, the number of experiments increases exponentially according to the number of heat sources. Im et al. [18] proposed an approach that uses both the linear superposition and RSM to overcome the disadvantages of each method. This composed method can calculate device junction temperature accurately and it requires much fewer experiments than RSM, especially in case of lots of heat sources.

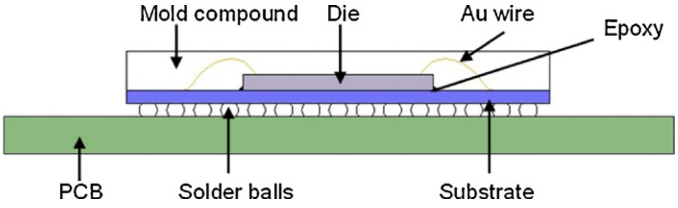

Fig. 1. Cross section of a typical BGA packaging bonded on PCB.

In this paper, an analytical thermal resistance network model for calculating the mean die temperature of typical plastic BGA packaging is presented. Unlike other thermal network models mentioned above, the model proposed in this paper does not need analysis tools. It also does not needs any experiments to evaluate mean die temperature. All of the resistances in the network have analytical solutions. Therefore, the mean die temperature can be calculated by programming based on the analytical solutions of thermal resistance in the network. The mean die temperatures of a typical plastic BGA packaging which are predicted by this model are close to the ones obtained by numerical simulations at various thermal conditions. This model is accurate enough for the semiconductor industry. This model can also be used to guide the thermal management design of plastic BGA packaging.

2. Model development

As shown in Fig. 1, a typical plastic BGA packaging contains mold compound, die, substrate, epoxy, solder balls and Au wire which is used to connect die and substrate. In steady-state, the heat generated by die dissipates to ambient mainly in two paths: (1) parts of heat flows from die to mold compound and then transfers to ambient from top surface and the edge of mold compound; and (2) the rest heat flows from die to substrate and then separates into two parts: one transfers to ambient directly from the area where is not covered by solder balls at the bottom surface of substrate, the other one conducts to solder balls and spreads into PCB, and then transfers to ambient from top and bottom surfaces of PCB respectively. According to the two paths of heat dissipation from die to ambient, the thermal resistance network is established as shown in Fig. 2.

R_{ma} includes the resistances of mold compound and the resistance between surfaces of mold compound and ambient. R_{sub} is spreading resistance and one-dimensional conductivity resistance of substrate. R_{suba} is resistance between exposed bottom surface of substrate and ambient. R_{b} is resistance of solder balls. R_{Pa} includes the resistance of PCB and the resistance between PCB surfaces and ambient.

The detailed analytical solution of every thermal resistance in Fig. 2 will be discussed in the following parts respectively.

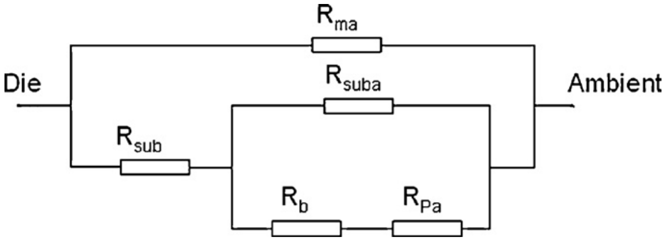

Fig. 2. Thermal resistance network of the typical BGA packaging shown in Fig. 1.

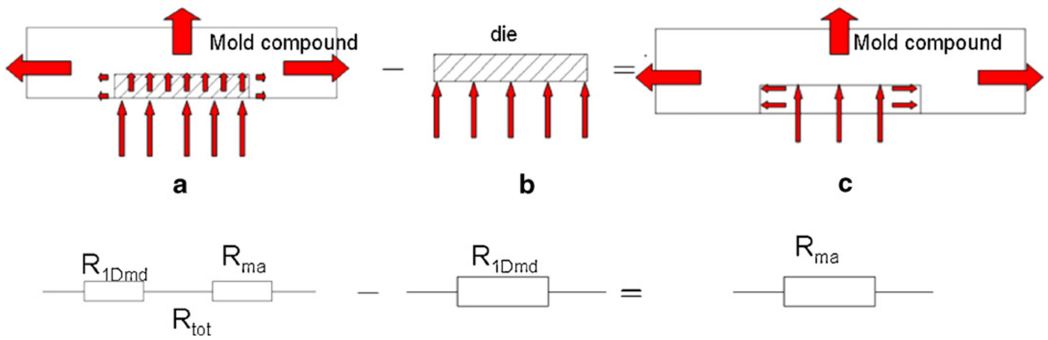

Fig. 3. Schematic diagram for calculating R_{ma}. (a) A rectangular flux channel of mold compound with die; (b) Mold compound with die size; and (c) Actual flux channel of mold compound.

2.1. Calculation of R_{ma}

The actual heat transfer in top part of the packaging is that heat generated by die spreads into mold compound and then dissipates to ambient as shown in Fig. 3(c). To calculate R_{ma}, as shown in Fig. 3(a), it is supposed that heat flows from the bottom of die and dissipates through the die, this heat transfer process is the same as actual situation. Therefore, as shown in Fig. 3, R_{ma} can be calculated by the following expression, $R_{\mathrm{ma}} = R_{\mathrm{tot}} - R_{\mathrm{1Dmd}}$, where R_{tot} is the total thermal resistance as heat flows from the bottom of die to ambient, R_{1Dmd} is one-dimensional thermal conduction resistance of die size mold compound.

Muzychka et al. [13] obtained a total thermal resistance expression for calculating total thermal resistance of rectangular flux channels with edge cooling. As the present case shown in Fig. 3(a) is as same as that in Ref. [13], thus R_{tot} can be calculated using the expression obtained by Muzychka et al. in Ref. [13]. The total resistance R_{tot} is calculated from the following general expression according to the notions in Fig. 4:

$$R_{\mathrm{tot}} = \frac{cd}{k_{\mathrm{m}} a^2 b^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin^2(\delta_{xm} a/c) \sin^2(\delta_{yn} b/d) \phi_{mn}}{\delta_{xm} \delta_{yn} \beta_{mn} [\sin(2\delta_{xm})/2 + \delta_{xm}][\sin(2\delta_{yn})/2 + \delta_{yn}]} \tag{5}$$

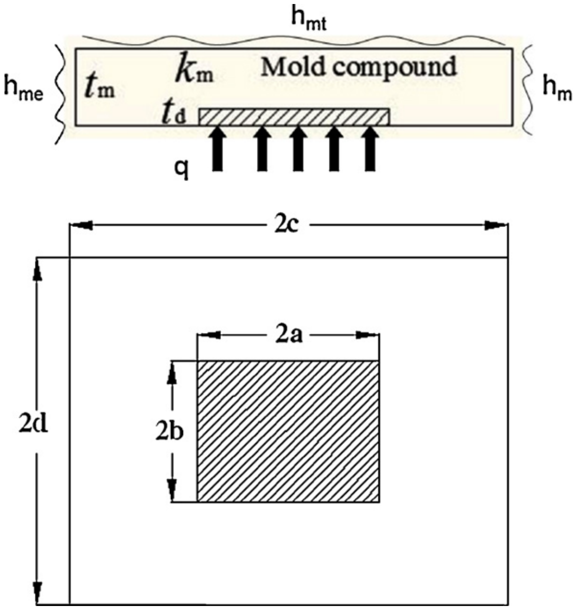

Fig. 4. Rectangular flux channel with edge cooling and relative dimensions, top figure: section view, bottom figure: bottom view.

where:

$$\delta_{xm} \tan(\delta_{xm}) = \frac{h_{\mathrm{me}} c}{k_m}, \quad \delta_{yn} \tan(\delta_{yn}) = \frac{h_{\mathrm{me}} d}{k_m} \tag{6}$$

$$\beta_{mn} = \sqrt{\left(\frac{\delta_{xm}}{c}\right)^2 + \left(\frac{\delta_{yn}}{d}\right)^2} \tag{7}$$

$$\phi_{mn} = \frac{\beta_{mn} t_m + ((h_{mt} t_m)/(k_m)) \tan h(\beta_{mn} t_m)}{((h_{mt} t_m)/(k_m)) + \beta_{mn} t_m \tan h(\beta_{mn} t_m)} \tag{8}$$

As shown in Fig. 3, R_{ma} is calculated as follows:

$$R_{\mathrm{ma}} = R_{\mathrm{tot}} - R_{\mathrm{1Dmd}} \tag{9}$$

where:

$$R_{\mathrm{1Dmd}} = \frac{t_d}{k_m 4ab} \tag{10}$$

Therefore, R_{ma} can be obtained by substituting Eqs. (5) and (10) into Eq. (9).

2.2. Calculation of R_{suba}

R_{suba} is the thermal resistance between the bottom surface of substrate and ambient. Here the bottom substrate is not covered by solder balls. R_{suba} is given by:

$$R_{\mathrm{suba}} = \frac{1}{h_{\mathrm{subbo}} A_{\mathrm{subbo}}} \tag{11}$$

where:

$$A_{\mathrm{subbo}} = 4cd - \frac{\pi D_1^2}{4} N \tag{12}$$

where N is the number of solder balls and D_1 will be given in Section 2.3.

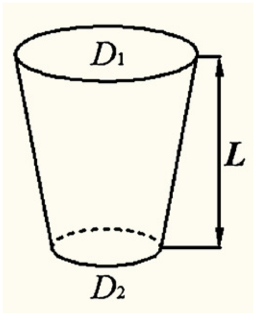

Fig. 5. Solder ball equivalent.

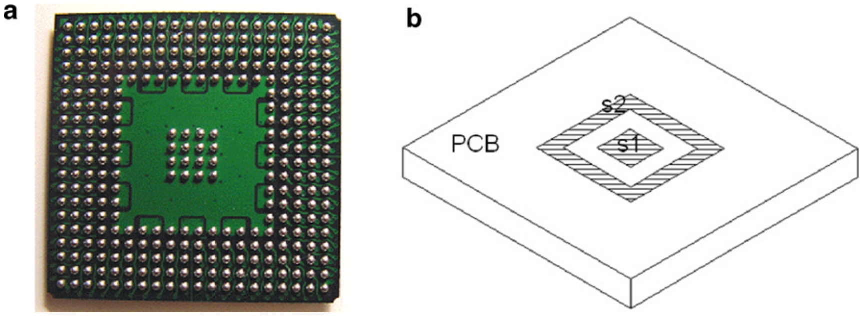

Fig. 6. (a) Solder ball array of a typical plastic BGA packaging; and (b) the areas where solder balls located on PCB (s1 and s2).

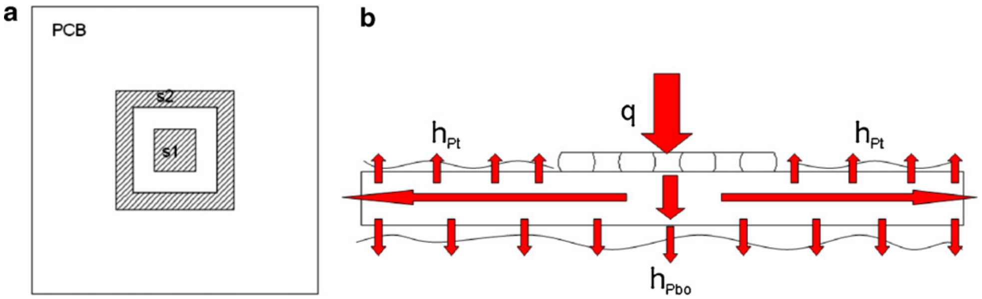

Fig. 7. (a) Simplified heat sources on PCB; and (b) heat dissipation paths through PCB.

2.3. Calculation of R_b

In the model, a solder ball is assumed to be equivalent to a truncated cone as shown in Fig. 5. Thermal resistance of the truncated cone is given by:

$$R_{b1} = \frac{4L}{\pi k_b D_1 D_2} \tag{13}$$

where D_1 and D_2 are diameters of circles at the top and bottom surfaces of truncated cone respectively, L is the height of truncated cone.

Heat flows through solder balls in parallel. Therefore, the total thermal resistance of solder balls is:

$$R_b = \frac{R_{b1}}{N} \tag{14}$$

2.4. Calculation of R_{Pa}

Ball array of a typical BGA packaging is shown in Fig. 6(a). Generally, array of balls at bottom of BGA packaging consists of two parts. One part of balls are arranged at the central area of the bottom. The rest of balls are arrayed around the central ones. Since solder balls are arranged to be close to each other, the areas where solder balls are bonded on can be simplified to heat sources as shaded parts shown in Fig. 6(b). The shaded parts in Fig. 6(b) are the areas where solder balls are located on PCB.

Heat dissipation paths through PCB are presented as follows. Heat coming from packaging flows into s1 and s2 as shown in Fig. 7(a), then it conducts and spreads onto PCB and finally transfers to ambient by convection from the top and bottom surfaces of PCB as shown in Fig. 7(b), respectively. According to the heat dissipation paths through PCB, the thermal resistance network for calculating R_{Pa} is established as Fig. 8. Since the PCB is very thin compared to its width and length, it is assumed that the heat flows from the solder balls only spreads along the horizontal directions and then dissipates to the ambient form top and bottom surfaces of the PCB. Thus the resistances in Fig. 8 are connected in a parallel way.

In Fig. 8, R_{Ps} is spreading resistance of PCB. R_{1DP} is one-dimensional thermal conduction resistance of PCB. R_{hPt} is thermal resistance between top of PCB, which is exposed to ambient. R_{hPbo} is the thermal resistance between PCB bottom part and ambient.

Dimensions of heat sources s1, s2 and PCB are shown in Fig. 9(a). Thermal parameters of PCB and boundary conditions are shown in Fig. 9(b).

2.4.1. Calculation of R_{hPt}

The area of top surface of PCB, which is exposed to ambient, is given by:

$$A_{Pt} = 4c_1 d_1 - 4cd \tag{15}$$

R_{hPt} is expressed as:

$$R_{hPt} = \frac{1}{h_{Pt} A_{Pt}} \tag{16}$$

2.4.2. Calculation of R_{hPbo}

The area of PCB's bottom surface is given by:

$$A_{Pbo} = 4c_1 d_1 \tag{17}$$

R_{hPbo} is expressed as:

$$R_{hPbo} = \frac{1}{h_{Pbo} A_{Pbo}} \tag{18}$$

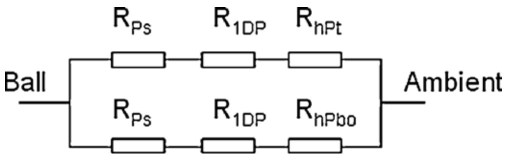

Fig. 8. Thermal resistances network used to calculate R_{Pa}.

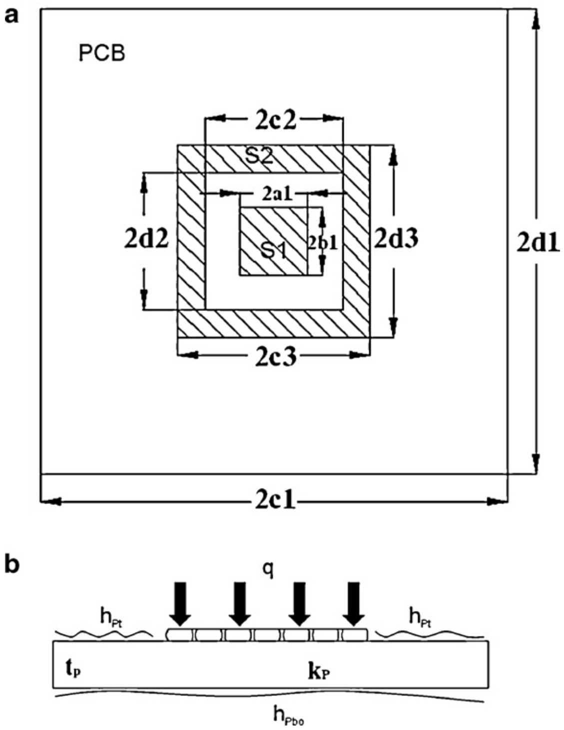

Fig. 9. (a) Dimensions of heat sources s1, s2 and PCB; and (b) thermal parameters of PCB and boundary conditions.

2.4.3. Calculation of R_{Ps} and R_{1DP}

For isotropic plate with single heat source on its surface, the total thermal resistance is defined as:

$$R_{\text{total}} = \frac{\bar{T}_{\text{source}} - T_{\text{a}}}{Q} \tag{19}$$

where $\bar{T}_{\text{source}}$ is mean sources temperature given by:

$$\bar{T}_{\text{source}} = \frac{1}{A_{\text{source}}} \int\int_{A_{\text{source}}} T(x, y, 0) dA_{\text{source}} \tag{20}$$

where A_{source} is the source area, $(x, y, 0)$ is coordinate of the top surface and 0 is z direction component.

Muzychka et al. [12] obtained the general solution for temperature distribution of isotropic plate, on which single rectangular heat source is located, with top and edge boundaries adiabatic. For multiple sources, the temperature distribution of isotropic plate is obtained by superposition [12].

Heat sources s1 and s2 are separated into five rectangular heat sources as shown in Fig. 10(a) for applying the solution obtained in Ref. [12]. So the temperature distribution on top surface of PCB can be achieved by superposition as:

$$T(x, y, 0) - T_{\text{a}} = \sum_{i=1}^{5} \theta_i(x, y, 0) \tag{21}$$

where θ_i is the temperature rise for each heat source by itself. In the model, the PCB is supposed to be mounted on a heat sink. Heat transfer rate at bottom surface of the PCB is much more than the one at top surface. As a result, the equivalent heat transfer coefficient at bottom surface of the PCB will be much more than the one at top surface where nature convection happens. Thus the top surface of the PCB can be considered to be adiabatic so that R_{Ps} and R_{1DP} are assumed to be only relevant with heat transfer coefficient at bottom surface of the PCB h_{pbo}. For those cases that heat transfer coefficient at top surface of PCB is close to or even larger than the one at bottom surface, R_{Ps} and R_{1DP} should be calculated in another method, which is represented in Ref. [19]. As the thickness of PCB is much less than its width and length, edges of PCB are also treated as adiabatic. Therefore, θ_i can be obtained using the general solution for temperature distribution of isotropic plate with adiabatic conditions at boundaries. θ_i is given by:

$$\theta_i(x, y, 0) = Q^i\Big(A_0^i + \sum_{m=1}^{\infty} A_m^i \cos(\lambda x) + \sum_{n=1}^{\infty} A_n^i \cos(\delta y) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^i \cos(\lambda x)\cos(\delta y)\Big) \tag{22}$$

Having obtained temperature distribution of top surface of PCB, the mean temperature rise of heat source s1 and s2 can be calculated by integrating Eq. (21) over the regions s1 and s2. However, it is complex to calculate the mean temperature rise because of the special shape of s2. To simplify the calculation, the mean temperature rise of rectangular region consisted by s1, s2 and the area between them substitutes the mean temperature rise of heat sources s1 and s2 in the present model. As temperature of this area is close to s1 and s2, this simplified treatment will not bring unacceptable error. The mean temperature rise of the substitute rectangular area is given by:

$$\overline{\theta} = \sum_{i=1}^{5} \frac{1}{A_{\text{substitute}}} \int\int_{A_{\text{substitute}}} \theta_i(x, y, 0) dA_{\text{substitute}} = \sum_{i=1}^{5} \overline{\theta_i} \tag{23}$$

where $A_{\text{substitute}} = 4c_3d_3$ and:

$$\overline{\theta_i} = Q^i \left(A_0^i + 2\sum_{m=1}^{\infty} A_m^i \frac{\cos(\lambda x_{\text{BGA}})\sin(\lambda c_3)}{2\lambda c_3} + 2\sum_{n=1}^{\infty} A_n^i \frac{\cos(\delta y_{\text{BGA}})\sin(\delta d_3)}{2\delta d_3} + 4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^i \frac{\cos(\delta y_{\text{BGA}})\sin(\delta d_3)\cos(\lambda x_{\text{BGA}})\sin(\lambda c_3)}{4\lambda\delta c_3 d_3} \right) \tag{24}$$

where Q_i is the heat flow of ith heat source, $(x_{\text{BGA}}, y_{\text{BGA}})$ is substitute rectangular centroid and:

$$A_0^i = \frac{1}{4c_1d_1}\left(\frac{t_{\text{P}}}{k_{\text{P}}} + \frac{1}{h_{\text{Pbo}}}\right) \tag{25}$$

$$A_m^i = \frac{\cos(\lambda x_i)\sin((1/2)\lambda L_i)}{c_1 d_1 L_i k_{\text{P}} \lambda^2 \phi(\lambda)} \tag{26}$$

$$A_n^i = \frac{\cos(\delta y_i)\sin((1/2)\delta W_i)}{c_1 d_1 W_i k_{\text{P}} \delta^2 \phi(\delta)} \tag{27}$$

$$A_{mn}^i = \frac{4\cos(\lambda x_i)\sin((1/2)\lambda L_i)\cos(\delta y_i)\sin((1/2)\delta W_i)}{c_1 d_1 L_i W_i k_{\text{P}} \beta\lambda\delta\phi(\beta)} \tag{28}$$

where $\lambda = \frac{m\pi}{2c_1}$, $\delta = \frac{n\pi}{2d_1}$, $\beta = \sqrt{\lambda^2 + \delta^2}$, and:

$$\phi(\xi) = \frac{\xi \sin h(\xi t_{\text{P}}) + h_{\text{Pbo}}/k_{\text{P}} \cos h(\xi t_{\text{P}})}{\xi \cos h(\xi t_{\text{P}}) + h_{\text{Pbo}}/k_{\text{P}} \sin h(\xi t_{\text{P}})} \tag{29}$$

ξ is replaced by λ, δ, or β, accordingly. (x_i, y_i) is ith source centroid. L_i and W_i are the length and width of ith source respectively. These parameters are given by:

$$x_1 = c_1 - \frac{c_2 + c_3}{2}, \quad x_2 = c_1 + \frac{c_2 + c_3}{2} \tag{30}$$

$$x_3, x_4, x_5 = c_1 \tag{31}$$

$$y_1, y_2, y_5 = d_1 \tag{32}$$

$$y_3 = d_1 - \frac{d_2 + d_3}{2}, \quad y_4 = d_1 + \frac{d_2 + d_3}{2} \tag{33}$$

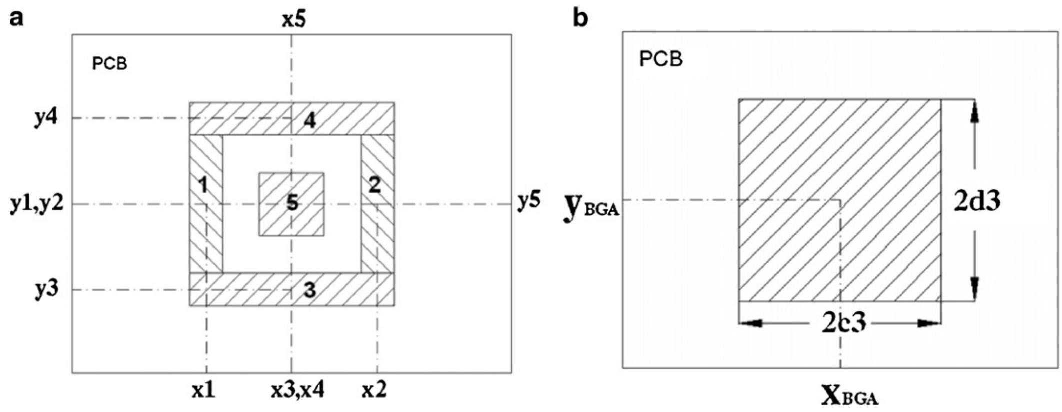

Fig. 10. (a) Heat sources on top surface of PCB; and (b) substitute rectangular area.

$$L_1, L_2 = c_3 - c_2, \quad W_1, W_2 = 2d_2 \tag{34}$$

$$L_3, L_4 = 2c_3, \quad W_3, W_4 = d_3 - d_2 \tag{35}$$

$$L_5 = 2a_2, \quad W_5 = 2b_2 \tag{36}$$

As $\overline{\theta}$ has been obtained, the total thermal resistance as defined by Eq. (19) is:

$$R_{\mathrm{total}} = \frac{\overline{\theta}}{Q} = R_{\mathrm{Ps}} + R_{\mathrm{1DP}} + R_{\mathrm{hPbo}} \tag{37}$$

where Q is heat flow rate of heat source s1 and s2. It is assumed that heat flux of s1 and s2 is constant q. Therefore, Q_i and Q are given by:

$$Q_i = L_i W_i q, \quad Q = q \sum_{i=1}^{5} L_i W_i \tag{38}$$

R_{Ps} and R_{1DP} are obtained by summary of Eqs. (23), (24), (37) and (38) as follows:

$$\begin{aligned} R_{\mathrm{Ps}} + R_{\mathrm{1DP}} = \sum_{i=1}^{5} \frac{L_i W_i}{\sum_{i=1}^{5} L_i W_i} \Big(& A_0^i + 2\sum_{m=1}^{\infty} A_m^i \frac{\cos(\lambda x_{\mathrm{BGA}})\sin(\lambda c_3)}{2\lambda c_3} \\ & + 2\sum_{n=1}^{\infty} A_n^i \frac{\cos(\delta y_{\mathrm{BGA}})\sin(\delta d_3)}{2\delta d_3} \\ & + 4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^i \frac{\cos(\delta y_{\mathrm{BGA}})\sin(\delta d_3)\cos(\lambda x_{\mathrm{BGA}})\sin(\lambda c_3)}{4\lambda\delta c_3 d_3} \Big) - R_{\mathrm{hPbo}} \end{aligned} \tag{39}$$

2.4.4. *Summary of R_{Pa} calculation*

As all resistances in the network shown in Fig. 8 have been calculated, R_{Pa} is expressed as:

$$R_{\mathrm{Pa}} = \frac{1}{1/(R_{\mathrm{Ps}} + R_{\mathrm{1DP}} + R_{\mathrm{hPt}}) + 1/(R_{\mathrm{Ps}} + R_{\mathrm{1DP}} + R_{\mathrm{hPbo}})} \tag{40}$$

2.5. *Calculation of R_{sub}*

In paper [12], Muzychka et al. obtained a spreading resistance expression for calculating spreading resistance of isotropic plate with central heat source and edge adiabatic. In present case, the substrate is thin compared to its length and width. Most of heat coming from die conducts to solder balls and transfers to ambient by convection at the bottom surface of the substrate. Therefore, the edge of substrate can be considered to be adiabatic. And die is located at central of substrate. R_{subs} can therefore be calculated by the spreading resistance expression obtained in Ref. [12]. Because there exists both conduction and convection at the bottom surface of substrate, the expression mentioned above cannot be used to calculate R_{subs} directly. To obtain R_{subs}, an equivalent heat transfer coefficient which is equivalent to actual heat transfer at bottom surface of the substrate should be found. It is supposed that heat transfers from substrate to ambient by convection with heat transfer coefficient h_{equ}. The total heat flowing from substrate is provided by:

$$q_{\mathrm{suba}} = h_{\mathrm{equ}} A_{\mathrm{subb}} (\bar{T}_{\mathrm{subb}} - T_{\mathrm{a}}) \tag{41}$$

Actually, according to thermal resistances network as shown in Fig. 2, the total heat flowing from substrate is:

$$q_{\mathrm{suba}} = \frac{\bar{T}_{\mathrm{subb}} - T_{\mathrm{a}}}{1/((1/R_{\mathrm{suba}}) + 1/(R_{\mathrm{b}} + R_{\mathrm{Pa}}))} \tag{42}$$

Eliminating q_{suba} in Eqs. (41) and (42), h_{equ} is obtained as:

$$h_{\mathrm{equ}} = \frac{(1/R_{\mathrm{suba}}) + (1/(R_{\mathrm{b}} + R_{\mathrm{Pa}}))}{A_{\mathrm{subb}}} \tag{43}$$

As h_{equ} has been found, the spreading resistance of substrate R_{subs} is obtained from the following general expression which shows the explicit relationships with the geometric and thermal parameters of the system according to the notations in Fig. 11:

$$\begin{aligned} R_{\mathrm{subs}} = & \frac{1}{2a^2 c d k_{\mathrm{sub}}} \sum_{m=1}^{\infty} \frac{\sin^2(a\delta_m)}{\delta_m^3} \varphi(\delta_m) \\ & + \frac{1}{2b^2 c d k_{\mathrm{sub}}} \sum_{n=1}^{\infty} \frac{\sin^2(b\lambda_n)}{\lambda_n^3} \varphi(\lambda_n) \\ & + \frac{1}{a^2 b^2 c d k_{\mathrm{sub}}} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\sin^2(a\delta_m)\sin^2(b\lambda_n)}{\delta_m^2 \lambda_n^2 \beta_{mn}} \varphi(\beta_{mn}) \end{aligned} \tag{44}$$

where:

$$\delta_m = \frac{m\pi}{c}, \quad \lambda_n = \frac{n\pi}{d} \tag{45}$$

$$\beta_{mn} = \sqrt{\delta_m^2 + \lambda_n^2} \tag{46}$$

$$\varphi(\xi) = \frac{(e^{2\xi t_{\mathrm{sub}}} + 1)\xi - (1 - e^{2\xi t_{\mathrm{sub}}}) h_{\mathrm{equ}}/k_{\mathrm{sub}}}{(e^{2\xi t_{\mathrm{sub}}} - 1)\xi + (1 + e^{2\xi t_{\mathrm{sub}}}) h_{\mathrm{equ}}/k_{\mathrm{sub}}} \tag{47}$$

And R_{sub} is given by:

$$R_{\mathrm{sub}} = R_{\mathrm{subs}} + R_{\mathrm{1Dsub}} \tag{48}$$

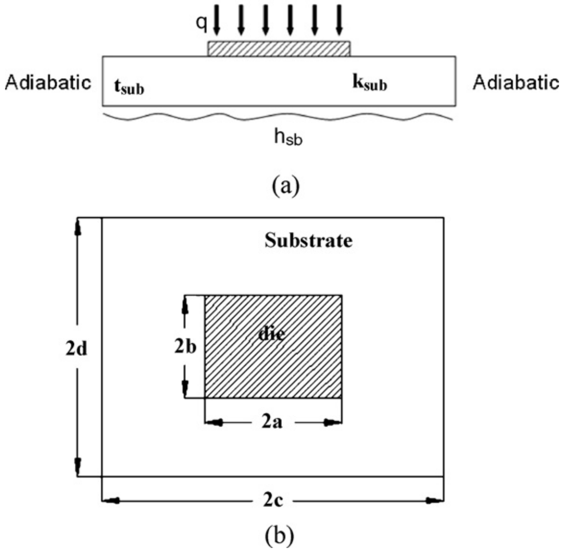

Fig. 11. Substrate with die located in center, top figure: section view, bottom figure: top view.

where:

$$R_{1\mathrm{Dsub}} = \frac{t_{\mathrm{sub}}}{k_{\mathrm{sub}}4cd} \tag{49}$$

Therefore, R_{sub} can be obtained by substituting Eqs. (44) and (49) into (48).

2.6. Calculation of R_{to}

Based on the above equations, the total thermal resistance between die and ambient is given by:

$$R_{\mathrm{to}} = \frac{1}{(1/R_{\mathrm{ma}}) + (1/(R_{\mathrm{sub}} + (1/((1/R_{\mathrm{suba}}) + (1/(R_{\mathrm{b}} + R_{\mathrm{Pa}}))))))} \tag{50}$$

2.7. Prediction of mean die temperature $\bar{T}_d$

Generally, the total thermal resistance between die and ambient is defined as:

$$R_{\mathrm{to}} = \frac{\bar{T}_{\mathrm{d}} - T_{\mathrm{a}}}{Q} \tag{51}$$

where $\bar{T}_{\mathrm{d}}$ is the mean temperature of die, Q is the power input into die. Therefore, $\bar{T}_{\mathrm{d}}$ is obtained as:

$$\bar{T}_{\mathrm{d}} = R_{\mathrm{to}}Q + T_{\mathrm{a}} \tag{52}$$

3. Analysis and calculation

To demonstrate the feasibility of the present model, the mean die temperatures of system containing a typical plastic BGA packaging and PCB under three series of thermal conditions were predicted by the proposed model respectively. Simulations to obtain the mean die temperatures of the system under the same thermal conditions as the prediction model were done by commercial software COMSOL. The accuracy of the presented model is proven by comparing the simulation data with the predicted ones.

Table 1
Dimensions of the system.

	Parameter	Symbol	Dimension
Component			
Mold compound	Length (mm)	$2c$	23
	Width (mm)	$2d$	23
	Thickness (mm)	t_{m}	1.22
Die	Length (mm)	$2a$	8
	Width (mm)	$2b$	8
	Thickness (mm)	t_{d}	0.25
Substrate	Length (mm)	$2c$	23
	Width (mm)	$2d$	23
	Thickness (mm)	t_{sub}	0.67
Solder balls	Thickness (mm)	L	0.46
	Diameter (mm)	–	0.70
	Cone diameter (mm)	$D1, D2$	0.52
	Ball number	N	233
	Ball array in center	–	5 × 5
	Pitch (mm)	–	1.27
PCB	Length (mm)	$2c_1$	76
	Width (mm)	$2d_1$	76
	Thickness (mm)	t_{P}	1
Simplified structure			
Heat source s1	Length (mm)	$2a_1$	5.78
	Width (mm)	$2b_1$	5.78
Heat source s2	Length 1 (mm)	$2c_2$	12
	Width 1 (mm)	$2d_2$	12
	Length 2 (mm)	$2c_3$	21
	Width 2 (mm)	$2d_3$	21

Table 2
Invariable thermal parameters in three series of thermal conditions.

Thermal parameter	Symbol	Quantity
Heat transfer coefficient at top of mold compound, W/(m^2 K)	h_{mt}	5
Heat transfer coefficient at edge of mold compound, W/(m^2 K)	h_{me}	5
Heat transfer coefficient at bottom of substrate exposed to ambient, W/(m^2 K)	h_{subbo}	1
Heat transfer coefficient at top of PCB, W/(m^2 K)	h_{Pt}	5
Thermal conductivity of mold compound, W/(m K)	k_{m}	0.2
Thermal conductivity of solder balls, W/(m K)	k_{b}	20
Ambient temperature (K)	T_{a}	293.15
Power input die (W)	Q	5

In the first series, thermal conductivity of substrate k_{sub} is changed, other parameters are constant. In the second series, thermal conductivity of PCB k_{P} is the only variable thermal parameter. In the last series, heat transfer coefficient at the bottom surface of PCB h_{Pbo} is variable while other thermal parameters remain to be constants. Besides k_{sub}, k_{P} and h_{Pbo}, other thermal parameters and dimensions of plastic BGA packaging and PCB are selected based on commonly used ones in semi-conductor industry. The dimensions of the system are the same in all three series and they are given in Table 1. The invariable thermal parameters in the three series of thermal conditions are shown in Table 2.

The three series of thermal conditions are presented in Table 3. In series I, thermal conductivity of substrate k_{sub} is changed from 1 to 10 W/(m K) and the step is 1 W/(m K). In series II, thermal conductivity of PCB k_{P} is variable from 1 to 10 W/(m K) and the step is 1 W/(m K) too. In series III heat transfer coefficient at bottom surface of PCB h_{Pbo} is varied from 100 to 1000 W/(m^2 K) and the step is 100 W/(m^2 K). In each series, only one thermal parameter is variable while the other ones are constant as shown in Table 3.

MATLAB was employed for the present calculation in three series of thermal conditions given above. In Eq. (5), (100) terms were used in double summation. In other equations which are used to calculate spreading resistance or temperature distribution 100 terms were used in each single summation and every double summation consists of 10 000 terms.

Table 3
Three series of thermal conditions.

Thermal conditions series	k_{sub},W/(m K)	k_{P},W/(m K)	h_{Pbo}, W/(m^2 K)
I	1–10	5	500
II	5	1–10	500
III	5	5	100–1000

Table 4
Comparison of data obtained by the present model and simulations in thermal conditions series I.

k_{sub}, W/(m K)	$\bar{T}_{\mathrm{dmodel}}$ (K)	$\bar{T}_{\mathrm{dsimulation}}$ (K)	$\bar{T}_{\mathrm{dmodel}} - \bar{T}_{\mathrm{dsimulation}}$ (K)	Error (%)
1	441.818	469.499	−27.681	−15.697
2	407.508	415.694	−8.186	−6.680
3	391.782	394.189	−2.407	−2.382
4	381.733	381.767	−0.034	−0.038
5	374.425	373.356	1.069	1.333
6	368.738	367.142	1.596	2.157
7	364.126	362.294	1.832	2.650
8	360.279	358.369	1.910	2.929
9	357.005	355.105	1.900	3.067
10	354.174	352.335	1.839	3.107

Table 5
Comparison of data obtained by the present model and simulations in thermal conditions series II.

k_{P}, W/(m K)	$\bar{T}_{\mathrm{dmodel}}$ (K)	$\bar{T}_{\mathrm{dsimulation}}$ (K)	$\bar{T}_{\mathrm{dmodel}} - \bar{T}_{\mathrm{dsimulation}}$ (K)	Error (%)
1	396.895	410.226	−13.331	−11.387
2	385.242	391.036	−5.794	−5.919
3	380.051	382.559	−2.508	−2.805
4	376.797	377.226	−0.429	−0.510
5	374.425	373.356	1.069	1.333
6	372.546	370.323	2.223	2.881
7	370.98	367.831	3.149	4.217
8	369.63	365.719	3.911	5.389
9	368.439	363.889	4.550	6.432
10	367.369	362.277	5.092	7.366

Table 6
Comparison of data obtained by the present model and simulations in thermal conditions series III.

h_{Pbo}, W/(m^2 K)	$\bar{T}_{\mathrm{dmodel}}$ (K)	$\bar{T}_{\mathrm{dsimulation}}$ (K)	$\bar{T}_{\mathrm{dmodel}} - \bar{T}_{\mathrm{dsimulation}}$ (K)	Error (%)
100	430.694	419.257	11.437	9.069
200	404.625	396.259	8.366	8.114
300	390.452	385.070	5.382	5.855
400	381.136	378.154	2.982	3.508
500	374.425	373.356	1.069	1.333
600	369.31	369.789	−0.479	−0.625
700	365.261	367.010	−1.749	−2.368
800	361.963	364.774	−2.811	−3.925
900	359.217	362.927	−3.710	−5.317
1000	356.893	361.371	−4.478	−6.564

COMSOL was employed for simulations. All of the simulations were done in the same three series of thermal conditions as the ones used in the proposed model for calculating the mean die temperature. To make simulations close to the actual heat transfer processes, heat transfer coefficient at edge of substrate and PCB were added on. Since it is nature convection from edge of substrate and PCB, the heat transfer coefficients were given as 5 W/(m^2 K).

4. Results and discussions

Tables 4–6 present the data obtained by the proposed model and simulations. And the difference between mean die temperatures calculated by the presented model and the ones obtained by simulations, errors are also shown in the tables. The error is defined as:

$$\text{error} = \frac{(\bar{T}_{\mathrm{dmodel}} - T_{\mathrm{a}}) - (\bar{T}_{\mathrm{dsimulation}} - T_{\mathrm{a}})}{\bar{T}_{\mathrm{dsimulation}} - T_{\mathrm{a}}} \times 100\% \tag{53}$$

From Tables 4–6, it is found that in most cases the error is less than ±10%. The maximum error is 15.697% in thermal conditions series I, k_{sub}, 1 W/(m K). Those thermal conditions with error more than ±10% are not commonly used in typical BGA packaging. Therefore, the present model can be used to predict mean die temperature of typical BGA packaging with good accuracy. As shown in Tables 4 and 6, mean die temperature decreases considerably as k_{sub} and h_{Pbo} increases. These trends give a guide that k_{sub} and h_{Pbo} are two key factors which take significance impact on mean die temperature of typical BGA packaging. Therefore, the present model provides an optimization method to choose materials and boundary conditions used in typical plastic BGA packaging for good thermal management.

5. Conclusions

An analytical thermal resistance network model for predicting the mean die temperature of a typical plastic BGA packaging is presented. The proposed model was applied to predict the mean die temperature of a typical BGA packaging under three series of thermal conditions. Comparing the data obtained by the presented model and simulations, it is found that the proposed model predicted the mean die temperature in good accuracy. As each resistance in the network has an analytical solution, the proposed model can help find what the key factors influencing mean die temperature in a typical plastic BGA packaging are. This provides an optimization method to typical plastic BGA design for good thermal management. The extensions to other packages are under way.

Acknowledgments

The authors would like to acknowledge the financial support in part by 973 Project of The Ministry of Science and Technology of China (2009CB320203), in part by New Century Excellent Talents Project of The Chinese Education Ministry (NCET-10-0387).

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